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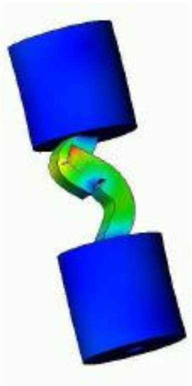

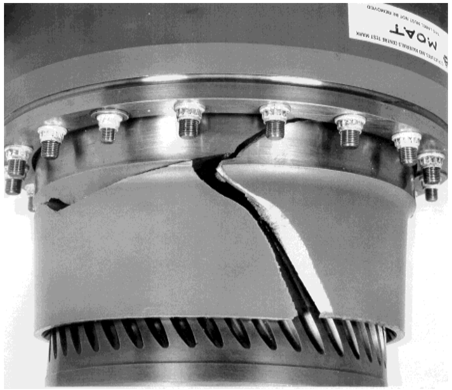

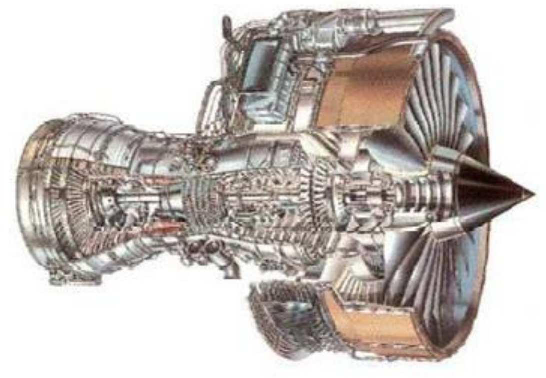

Project 3: Interface Reduction on Hurty/Craig-Bampton Substructures with Mechanical Joints

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July 27th, 2017

U.S. DEPARTMENT OF ENERGY

NNSA National Nuclear Security Administration

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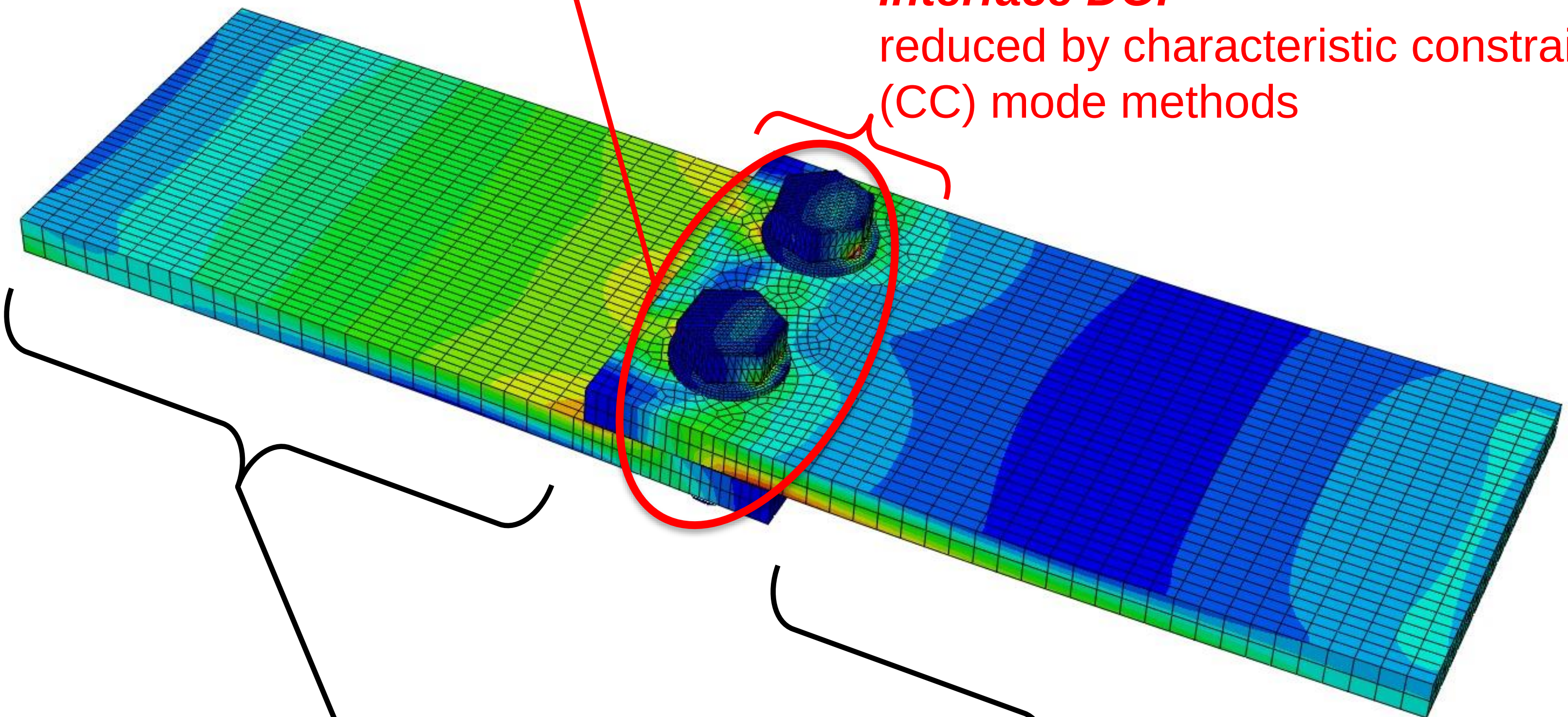
Agenda

- Background & motivation
- Theory Review
  - Hurty/Craig-Bampton substructuring (HCB method)
  - System-level characteristic constraint mode interface reduction (S_CC method)
  - Normal contact
  - Friction
- Selection of interface reduction basis
- Results
- Conclusions & future research

Background & motivation

Goal: add nonlinear elements here
& apply interface reduction

Interface DOF
reduced by characteristic constraint
(CC) mode methods

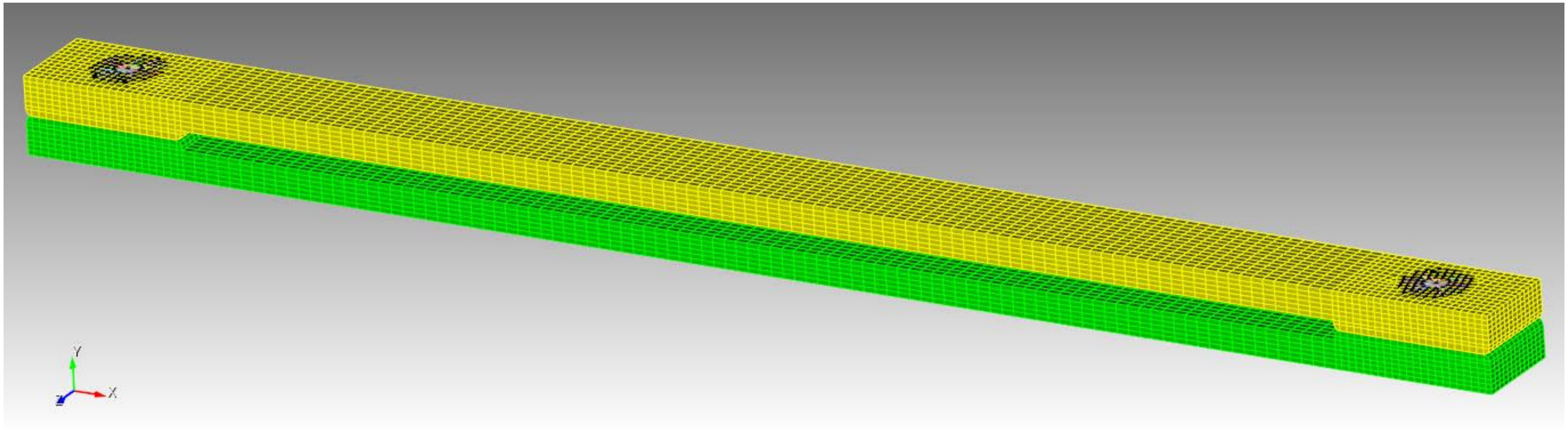


Interior DOF
reduced by component mode synthesis (CMS) methods

Image: http://www.ssanalysis.co.uk/blog/2012/10/29/bolted-joints-in-finite-element-models-our-first-training-webinar

Prototype C-beam assembly ("S4 beam")

- Analysis Overview
  - Full FEA Model (**94,000 DOF**) → HCB Model (**3,700 DOF**)
  - Define contact areas between surfaces with penalty spring elements
  - HCB Model (**3,700 DOF**) → SCC Model (**50 DOF**)
  - Use normal contact to define friction in contact plane
  - Simulate reduced order model and observe response



Review of Craig-Bampton Substructuring

- Equations of motion for an arbitrary dynamical system with localized nonlinearities

$$[\mathrm{M}]\{\ddot{\mathrm{u}}\} + [\mathrm{K}]\{\mathrm{u}\} + \{\mathrm{f}_{\mathrm{NL}}(\mathrm{u}, \dot{\mathrm{u}})\} = \{\mathrm{f}_{\mathrm{ext}}\}$$

- Apply Hurty/Craig-Bampton method to reduce interior (non-interface) degrees of freedom with $([\mathrm{M}_{\mathrm{ii}}] - \omega^2[\mathrm{K}_{\mathrm{ii}}])\ \Phi_{\mathrm{FI}} = \{0\}$

Where $\mathrm{n}_{\mathrm{u}_\mathrm{i}} \gg \mathrm{n}_{\mathrm{q}_\mathrm{i}}$

$$\begin{Bmatrix} \mathrm{u}_\mathrm{i} \\ \mathrm{u}_\mathrm{j} \end{Bmatrix} = \begin{bmatrix} \phi_{\mathrm{FI}} & -\mathrm{K}_{\mathrm{ii}}^{-1}\mathrm{K}_{\mathrm{ij}} \\ 0 & \mathrm{I} \end{bmatrix} \begin{Bmatrix} \mathrm{q}_\mathrm{i} \\ \mathrm{u}_\mathrm{j} \end{Bmatrix} = [\mathrm{T}_{\mathrm{HCB}}]\{\mathrm{q}\}$$

- Transform equations of motion:

$$[\mathrm{T}_{\mathrm{HCB}}]^\mathrm{T}[\mathrm{M}][\mathrm{T}_{\mathrm{HCB}}]\{\ddot{\mathrm{q}}\} + [\mathrm{T}_{\mathrm{HCB}}]^\mathrm{T}[\mathrm{K}][\mathrm{T}_{\mathrm{HCB}}]\{\mathrm{q}\} + [\mathrm{T}_{\mathrm{HCB}}]^\mathrm{T}\{\mathrm{f}_{\mathrm{NL}}(\mathrm{u}, \dot{\mathrm{u}})\} = [\mathrm{T}_{\mathrm{HCB}}]^\mathrm{T}\{\mathrm{f}_{\mathrm{ext}}\}$$

$$[\mathbf{M}_{\mathbf{HCB}}]\{\ddot{\mathbf{q}}\} + [\mathbf{K}_{\mathbf{HCB}}]\{\mathbf{q}\} + \left\{\mathbf{f}_{\mathbf{NL}}^{\mathbf{HCB}}(\mathbf{u}, \dot{\mathbf{u}})\right\} = \left\{\mathbf{f}_{\mathbf{ext}}^{\mathbf{HCB}}\right\}$$

Model size can still be unacceptably large because of the number of DOF at substructure interfaces

Review of System Characteristic Constraint Interface Reduction

- Reduction method requires all subcomponents to be assembled together first (CMS). Then, can keep interior modal DOFs and reduce physical interface DOFs using the S_CC method:

$$\{q\} = \begin{Bmatrix} q_i \\ u_j \end{Bmatrix} = \begin{bmatrix} I & 0 \\ 0 & \Psi \end{bmatrix} \begin{Bmatrix} q_i \\ q_j \end{Bmatrix} = [T_{SCC}]\{s\}$$

with $\left(M_{jj} - \omega^2 K_{jj}\right)\Psi = \{0\}$

- Apply Transformation:

$$[T_{SCCe}]^T[M_{HCB}][T_{SCCe}]\{\ddot{s}\} + [T_{SCCe}]^T[K_{HCB}][T_{SCCe}]\{s\} + [T_{SCCe}]^T\{f_{NL}^{HCB}(u,\dot{u})\} = [T_{SCCe}]^T\{f_{ext}^{HCB}\}$$

$$[\mathbf{M}_{\mathbf{SCCe}}]\{\ddot{\mathbf{s}}\} + [\mathbf{K}_{\mathbf{SCCe}}]\{\mathbf{s}\} + \left\{\mathbf{f}_{\mathbf{NL}}^{\mathbf{SCCe}}(\mathbf{u},\dot{\mathbf{u}})\right\} = \{\mathbf{f}_{\mathbf{ext}}^{\mathbf{SCCe}}\}$$

Converts all remaining physical DOF to modal DOF

S_CC does not retain physical DOF

- Need physical DOF onto which we can apply preload:

Want to maintain physical bolt DOF:

$$\begin{bmatrix} \mathbf{q_i} \\ \mathbf{u_j} \end{bmatrix} \rightarrow \begin{bmatrix} \mathbf{q_i} \\ \mathbf{u_r} \\ \mathbf{u_b} \end{bmatrix} \rightarrow \begin{bmatrix} \mathbf{q_i} \\ \mathbf{q_r} \\ \mathbf{u_b} \end{bmatrix}$$

Retain physical DOF

And reduce such that: $\mathbf{n_{u_r}} \gg \mathbf{n_{q_r}}$

System Level Constraint Modes Expansion (SCCe)

$$M_{CB} = \begin{bmatrix} M_{CB_{ii}} & M_{CB_{ir}} & M_{CB_{ib}} \\ M_{CB_{ri}} & M_{CB_{rr}} & M_{CB_{rb}} \\ M_{CB_{bi}} & M_{CB_{br}} & M_{CB_{bb}} \end{bmatrix} \quad K_{CB} = \begin{bmatrix} \Omega_{FI}^2 & 0 & 0 \\ 0 & K_{CB_{rr}} & K_{CB_{rb}} \\ 0 & K_{CB_{br}} & K_{CB_{bb}} \end{bmatrix}$$

DOF Labels:
i = interior
b= bolt
r = remaining interface

$$\left(M_{CB_{rr}}\omega^2 - K_{CB_{rr}}\right)\psi_{SCC_{rr}} = 0$$

These modes aren't enough by themselves to correctly constrain the bolt and patch interfaces:

Augment system with constraint modes similar to the HCB method.

$$\Psi_{SCCe} = \left[\psi'_{SCC_{rr}} \quad \Phi_{CM}\right] \qquad \Phi_{CM} = \begin{bmatrix} -K_{rr}^{-1}K_{rb} \\ I_{n_b} \end{bmatrix}$$

Static Condensation

Then the transformation:

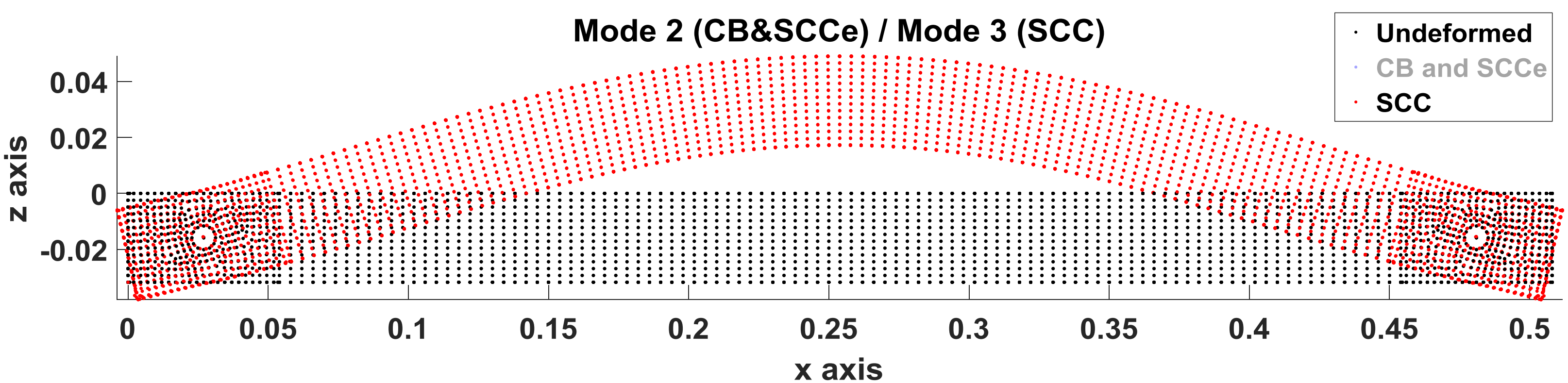
$$T_{SCCe} = \begin{bmatrix} I_{n_i} & 0 \\ 0 & \Psi_{SCCe} \end{bmatrix} = \begin{bmatrix} I_{n_i} & 0 & 0 \\ 0 & \psi_{SCC_{rr}} & -K_{rr}^{-1}K_{rb} \\ 0 & 0 & I_{n_b} \end{bmatrix} \qquad \begin{bmatrix} q_i \\ u_r \\ u_b \end{bmatrix} = T_{SCCe} \begin{bmatrix} q_i \\ q_r \\ u_b \end{bmatrix}$$

Expansion to S_CC Theory: SCCe

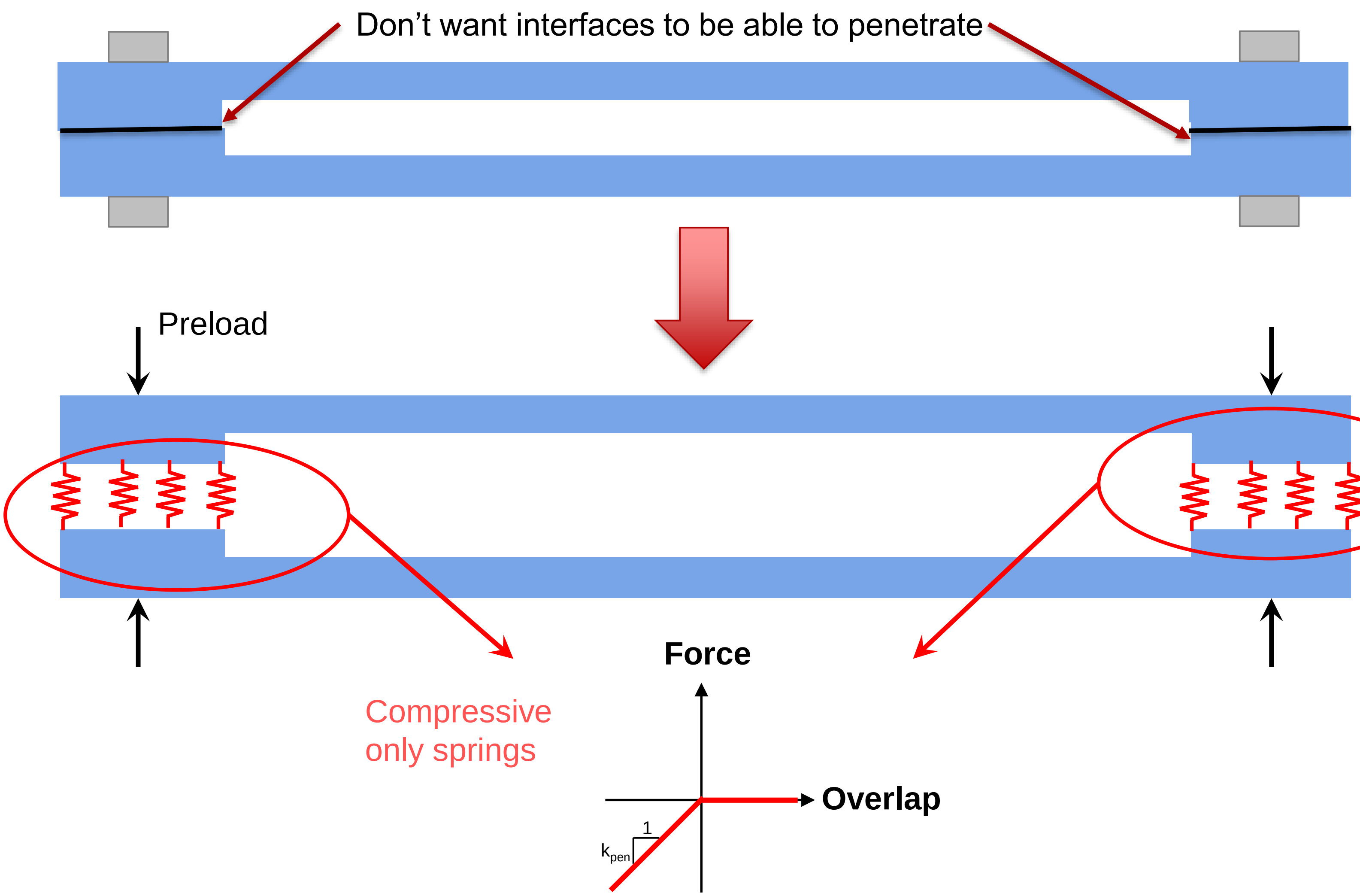
- Typically interface reduction means ALL of interface must be reduced
- Can't just multiply a partition of DOF by identity.
  - Causes reduced interface set to act like fixed interface modes.
  - Alleviate with constraint modes

$$T_{SCCe} = \begin{bmatrix} I_{n_i} & 0 \\ 0 & \Psi_{SCCe} \end{bmatrix} = \begin{bmatrix} I_{n_i} & 0 & 0 \\ 0 & \psi_{SCC_{rr}} & -K_{rr}^{-1}K_{rb} \\ 0 & 0 & I_{n_b} \end{bmatrix}$$

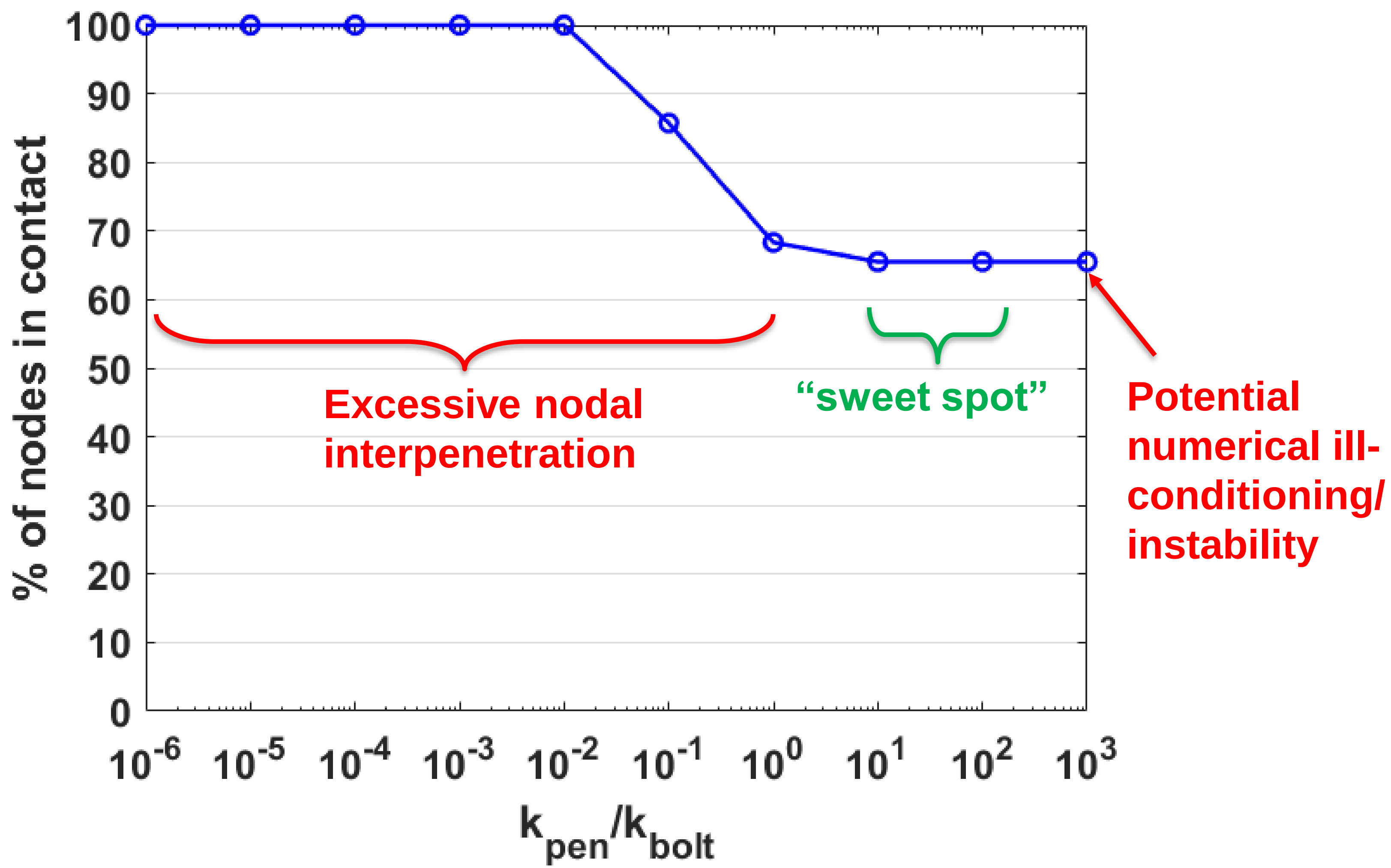
Physical DOF retained, 42 DOF model provides accuracy of <1% error for modes under 1kHz.



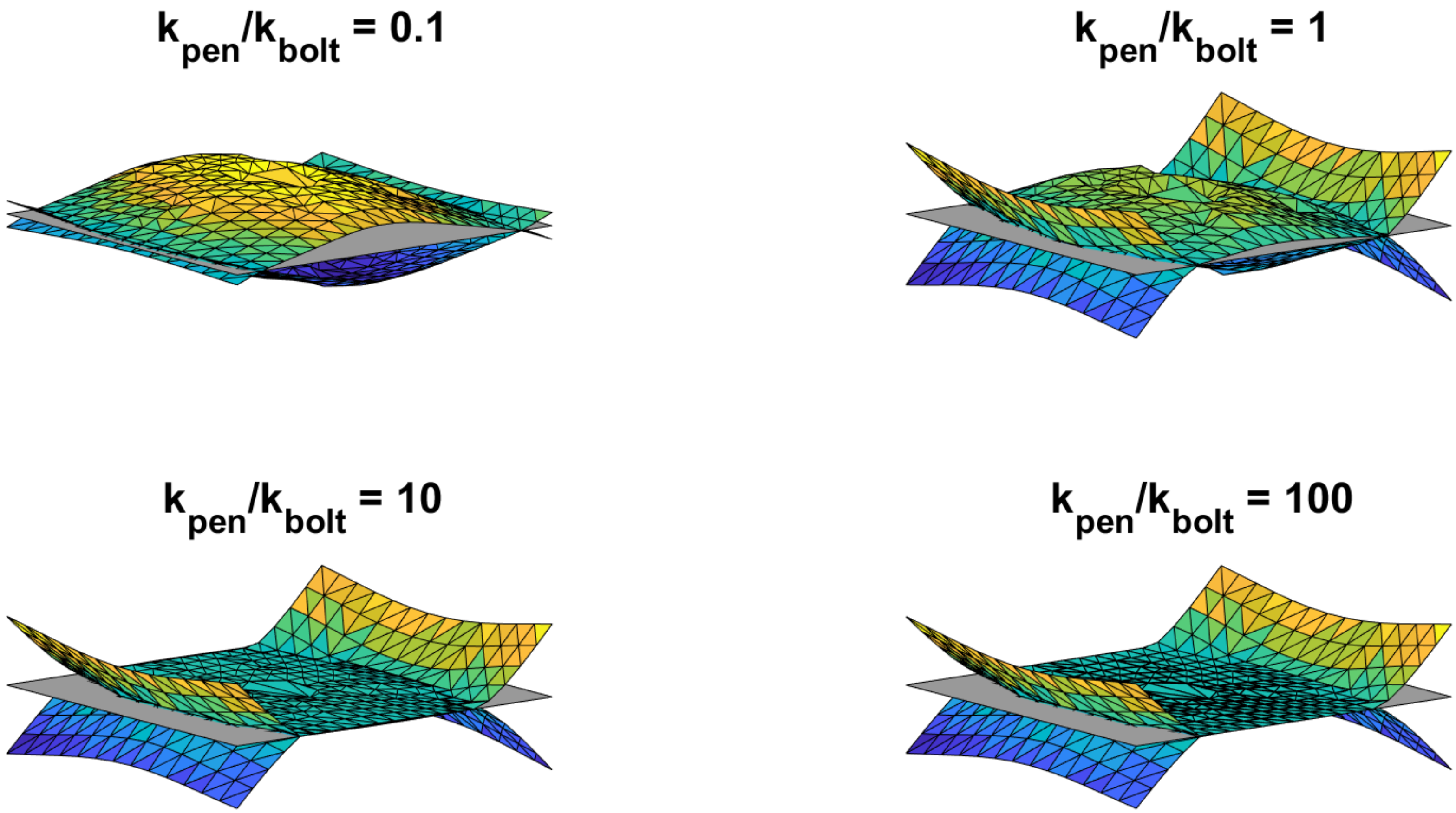
Normal contact model – penalty method



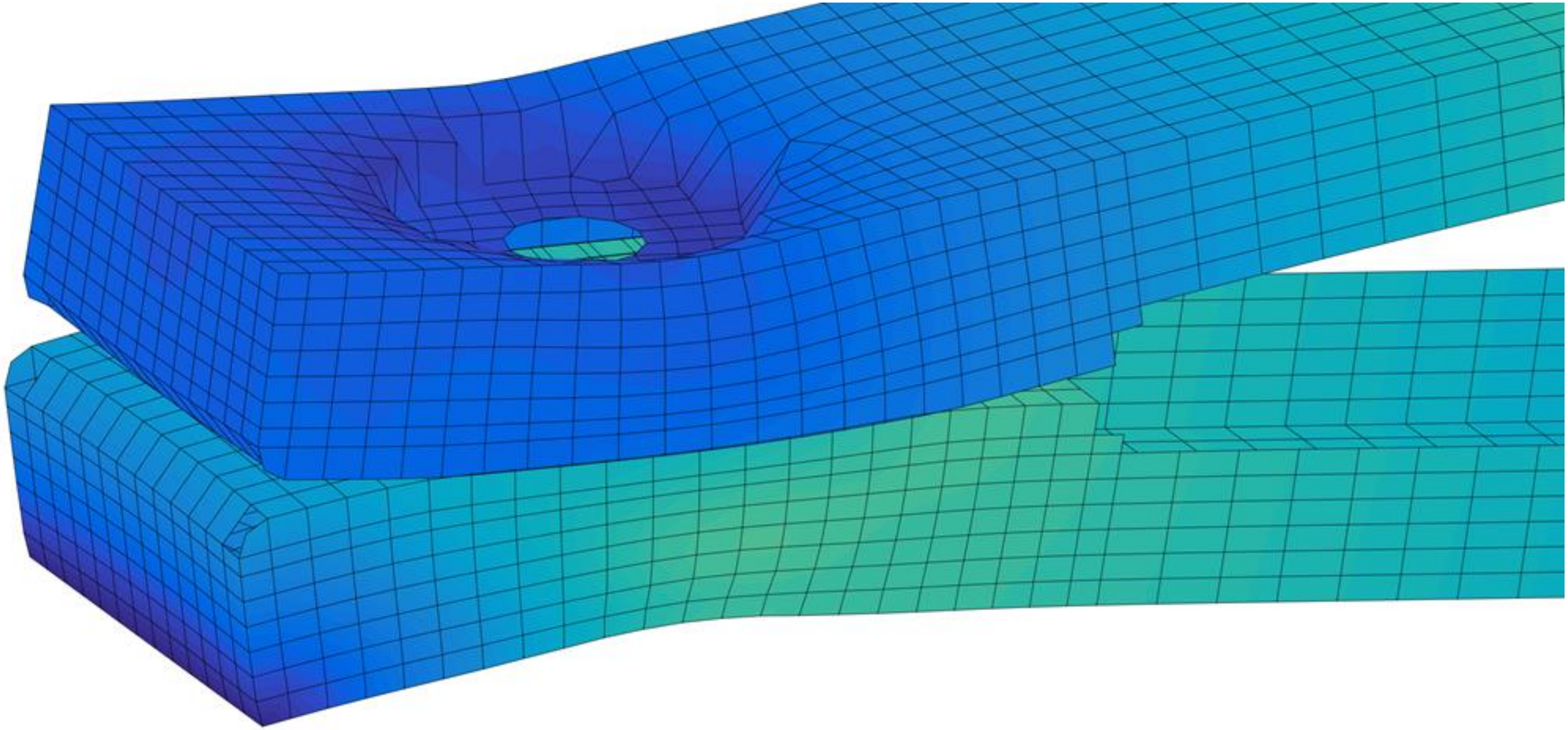
Sensitivity to penalty stiffness, k_{pen}



Sensitivity to penalty stiffness, k_{pen}



Preload-induced deformation ($k_{pen} = 100 \cdot k_{bolt}$)



Friction models

Coulomb's Law:

$$\mu(V_r) = \begin{cases} -\mu_d \,\mathrm{sign}(V_r) & \text{if } V_r \neq 0 \ \ \text{(slip)} \\ \mu_0 \ \text{ with } |\mu_0| \leq \mu_s, & \text{if } V_r = 0 \ \ \text{(stick)} \end{cases}$$

μ_S = static friction coefficient

μ_D = dynamic friction coefficient

V_r = relative velocity of contacting surfaces

Regularized Friction Model:

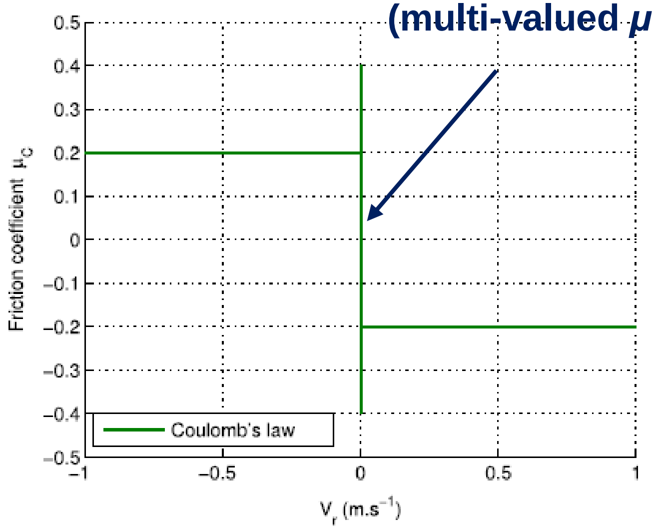
$$\mu_n(V_r) := g(nV_r) = \frac{-\mu_d V_r \sqrt{V_r^2 + \frac{\varepsilon}{n^2}} - 2\frac{\alpha}{n} V_r}{V_r^2 + \frac{1}{n^2}}$$

$$\alpha = \sqrt{\mu_s(\mu_s - \mu_d)}$$

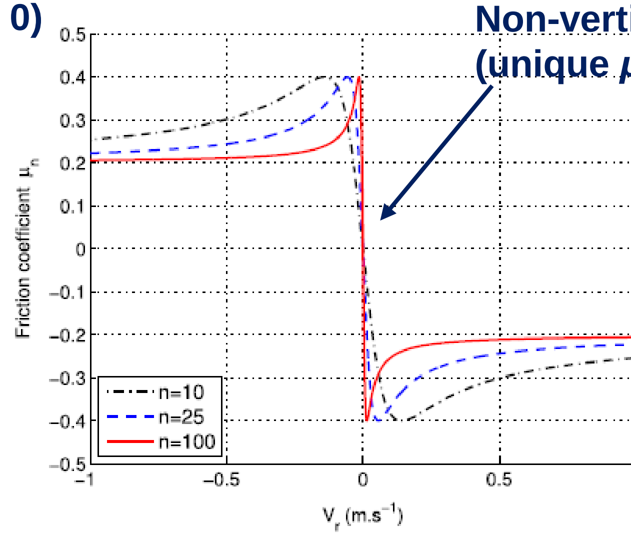
ε = model parameter (usually a small number ~ 10^{-4})

n = model parameter controlling stiffness of governing ODE (high $n \rightarrow$ stiffer system)

Perfectly vertical slope (multi-valued μ at $V_r = 0$)



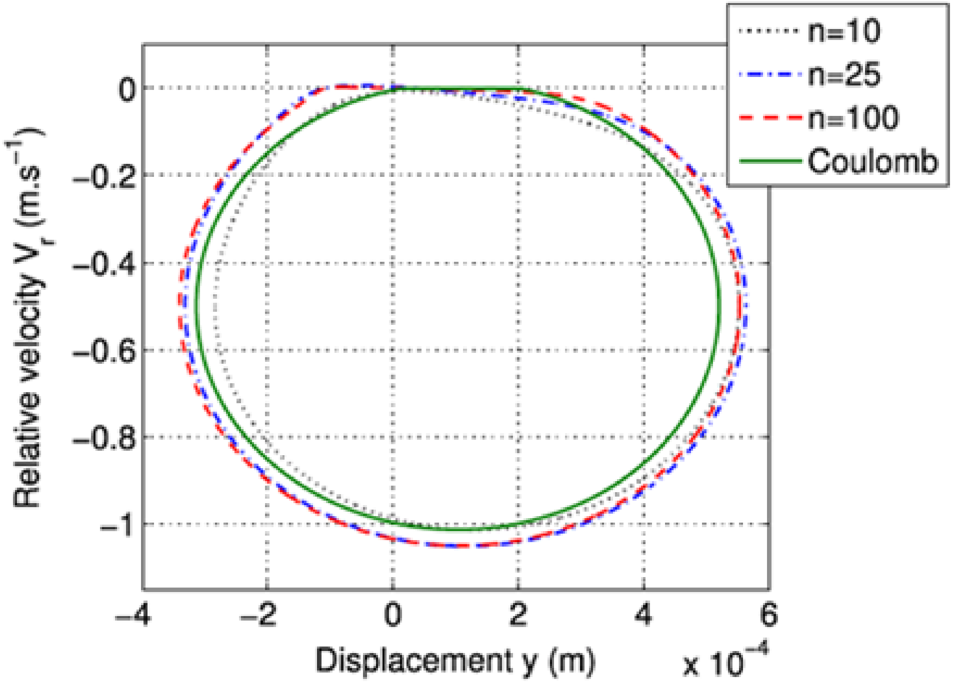
Non-vertical slope (unique μ for every V_r)



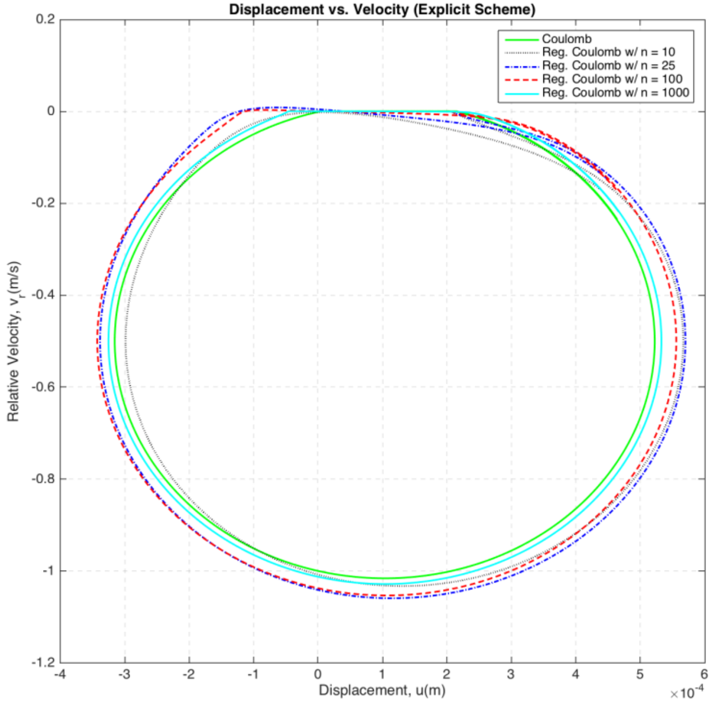
Verification: Regularized Coulomb friction models

Analytical solution:

Numerical solution:



Vigué, Pierre, et al. "Regularized friction and continuation: Comparison with Coulomb's law." *Journal of Sound and Vibration* 389 (2017): 350-363.



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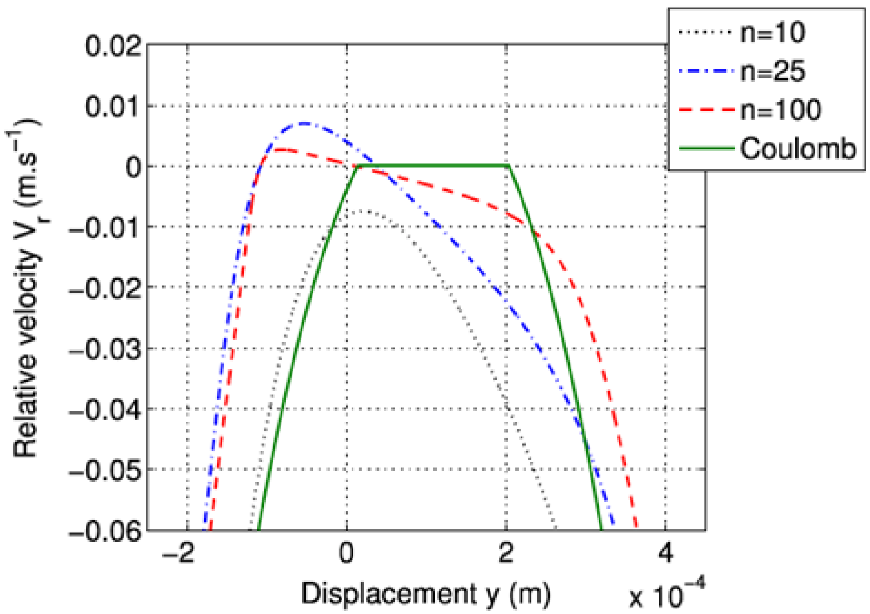
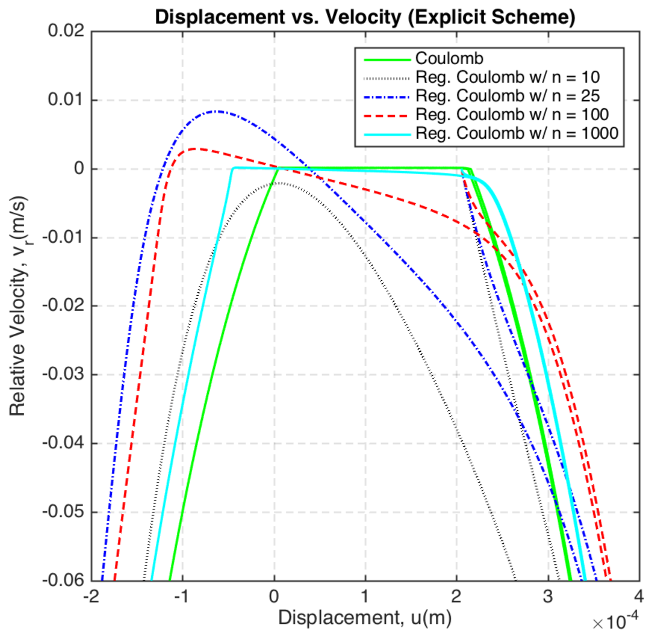
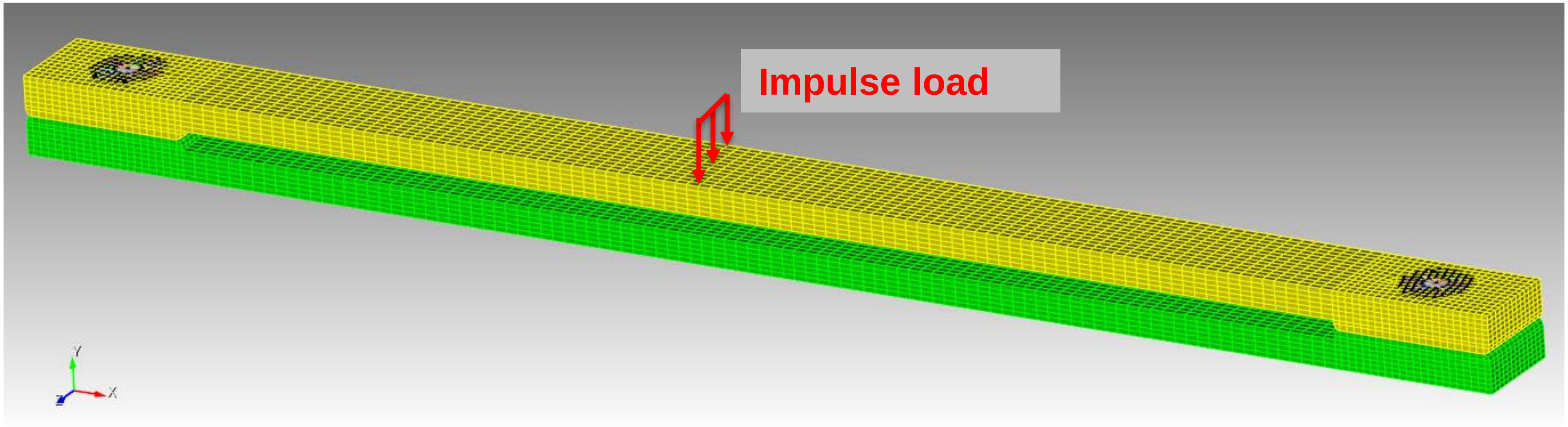


Numerical solution:

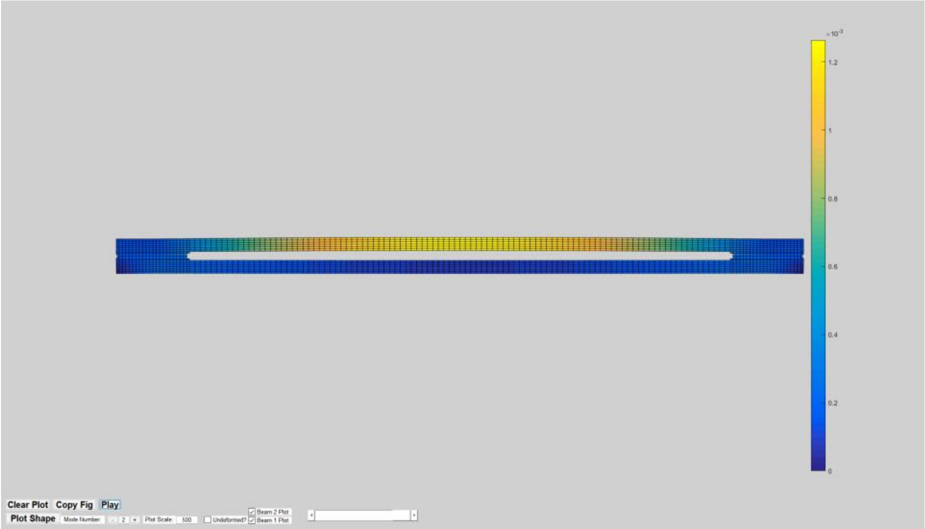


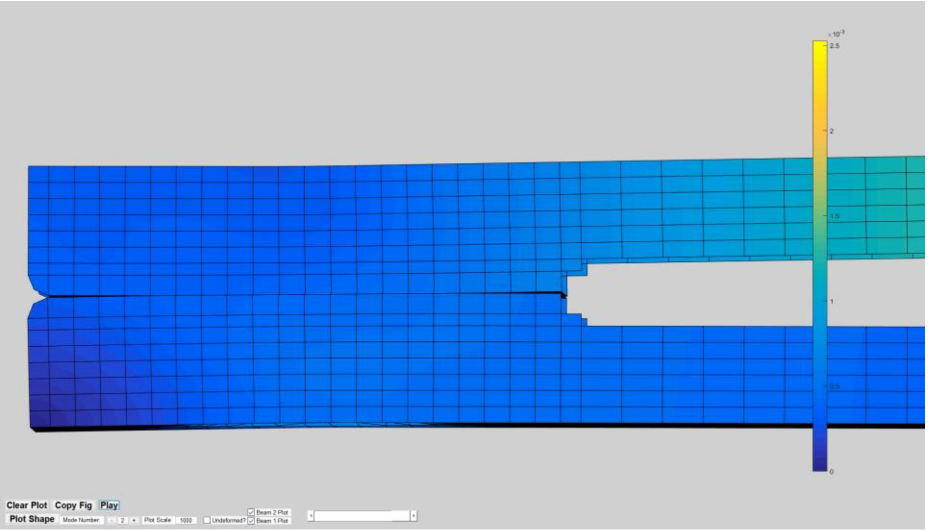
Vigué, Pierre, et al. "Regularized friction and continuation: Comparison with Coulomb's law." *Journal of Sound and Vibration* 389 (2017): 350-363.

HCB Results



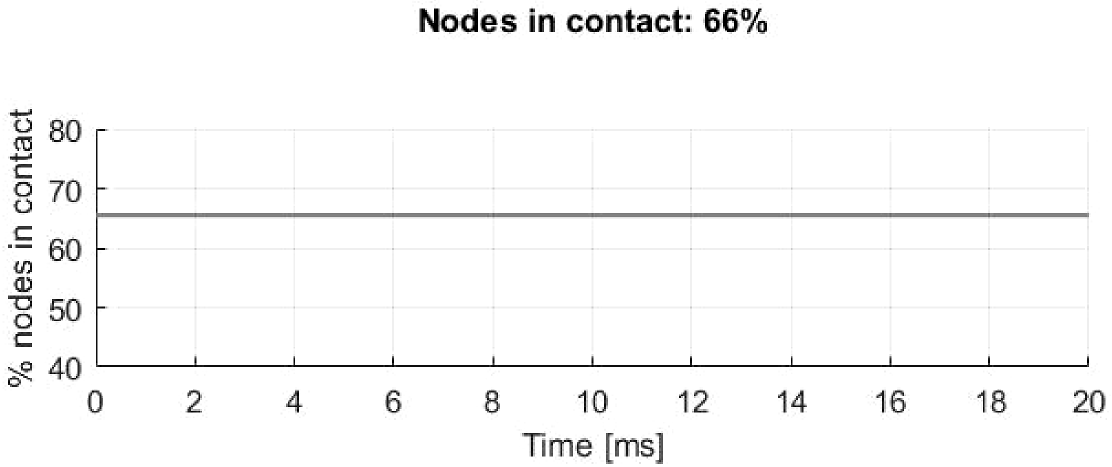
HCB Results – Full-field Deformation History





HCB Results - Time-evolution of contact area

Problem: *how do we choose the "right" characteristic constraint (CC) modes to capture the local dynamics at the interfaces?*



Selection of Interface Reduction Basis

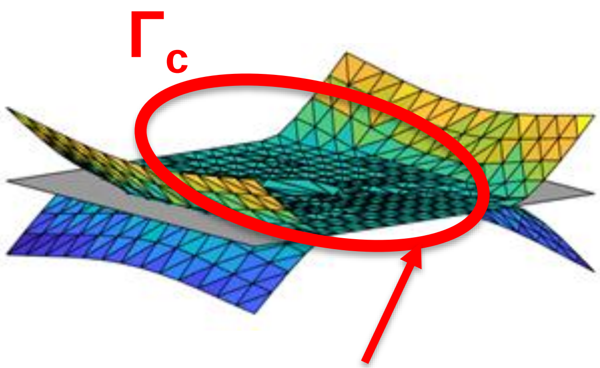
- Essence of the problem:
  - Change in system stiffness is governed by change in interface contact area (***nodes free to connect & disconnect***)
  - Interface-substructure force interaction is controlled by contacting nodes (***nodes constrained together***)
  - Need mode shapes that represent BOTH **free-interface** and **constrained-interface** motion

- Solution: constraining/unconstraining process to build mode shapes

Constrained/Unconstrained Mode Shapes

Perform preload analysis and determine:

- set of nodes in contact $\boldsymbol{\Gamma_c}$
- vector of nodal displacements $\{\mathbf{x_p}\}$



Build transformation matrix **[L]** that constrains node pairs in $\boldsymbol{\Gamma_c}$ to have the same y-displacement

$$\underbrace{\{u_{HCB}\}_u}_{\text{Nodes free to move independently}} = [L]\underbrace{\{u_{HCB}\}_c}_{\text{Nodes in } \Gamma_c \text{ partially constrained}}$$

$$[M_c] = [L]^T[M_{HCB}][L]$$

$$[K_c] = [L]^T[K_{HCB}][L]$$

Now have constrained M and K

Constrained/Unconstrained Mode Shapes

Build [Tc] using the SCCe method on **constrained system**

$$M_c = \begin{bmatrix} M_{c_{ii}} & M_{c_{ir}} & M_{c_{ib}} \\ M_{c_{ri}} & M_{c_{rr}} & M_{c_{rb}} \\ M_{c_{bi}} & M_{c_{br}} & M_{c_{bb}} \end{bmatrix}$$

$$K_c = \begin{bmatrix} \Omega_{FI}^2 & 0 & 0 \\ 0 & K_{c_{rr}} & K_{c_{rb}} \\ 0 & K_{c_{br}} & K_{c_{bb}} \end{bmatrix}$$

$$\left(M_{c_{rr}}\omega^2 - K_{c_{rr}}\right)\psi_{SCC_{rr}} = 0$$

$$T_c = \begin{bmatrix} I_{n_i} & 0 & 0 \\ 0 & \psi_{SCC_{rr}} & -K_{c_{rr}}^{-1}K_{c_{rb}} \\ 0 & 0 & I_{n_b} \end{bmatrix}$$

Transform **[T$_c$]** back to unconstrained coordinates using **[L]** & augment with preloaded nodal displacements **{x$_p$}**

$$[T_u] = \left[\ [L][T_c] \quad \{x_p\}\ \right]$$

From preload

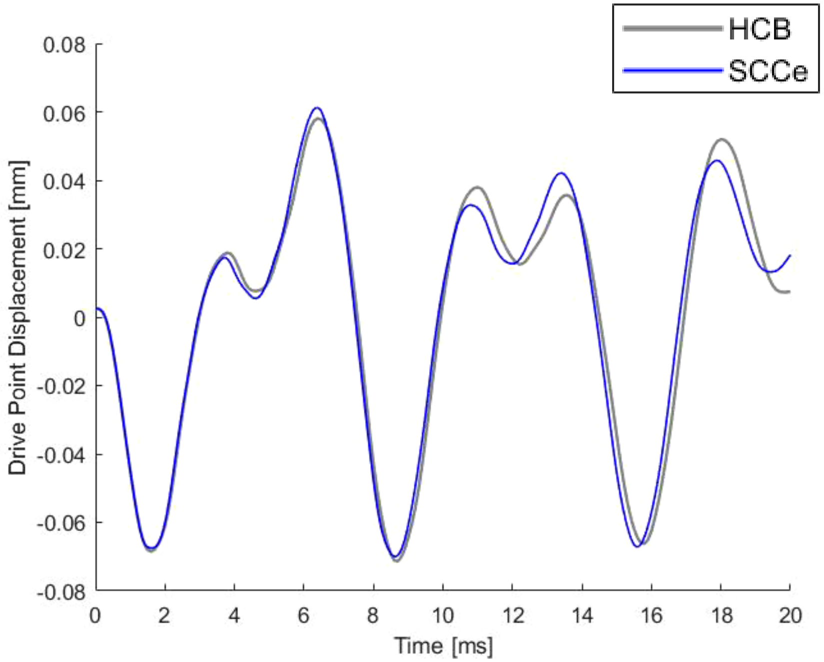
$$\{u_{SCCe}\} = [T_u]\{u_{HCB}\}_u$$

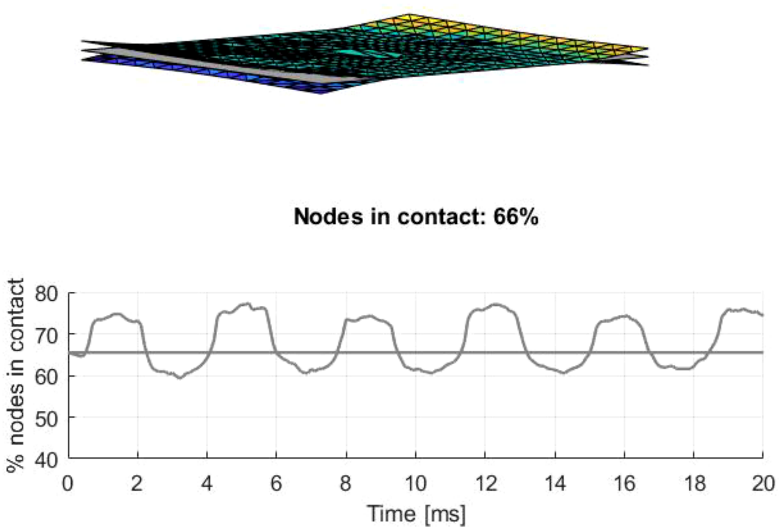
$$[M_{SCCe}] = [T_u]^T[M_{HCB}][T_u]$$

$$[K_{SCCe}] = [T_u]^T[K_{HCB}][T_u]$$

Can now use M_{SCCe} and K_{SCCe} to run dynamic analysis

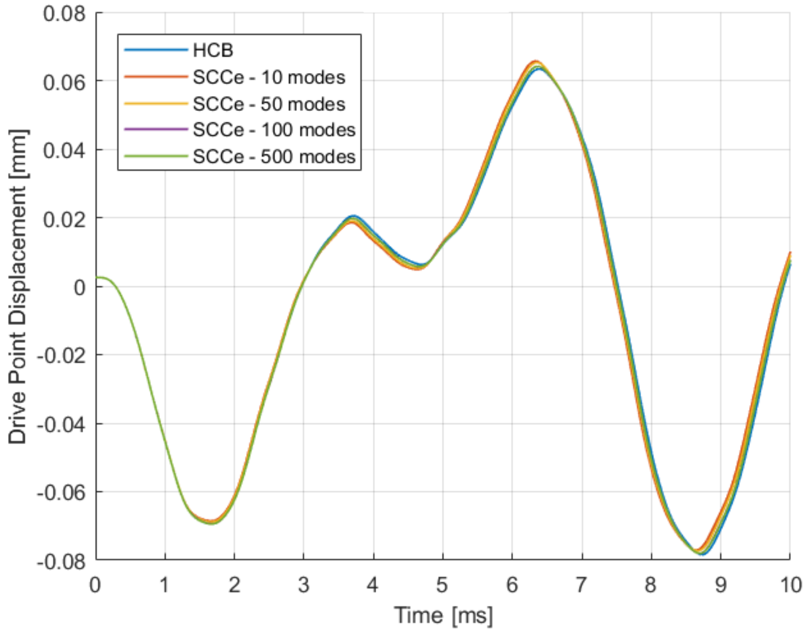
Results – 10 SCCe modes

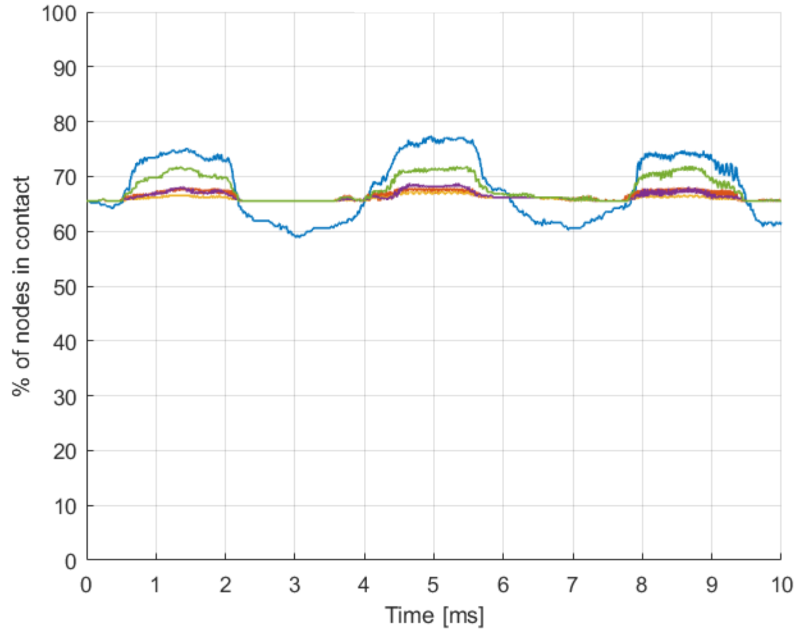




System-level displacement OK, but contact area not captured well

Results – 10+ SCCe modes

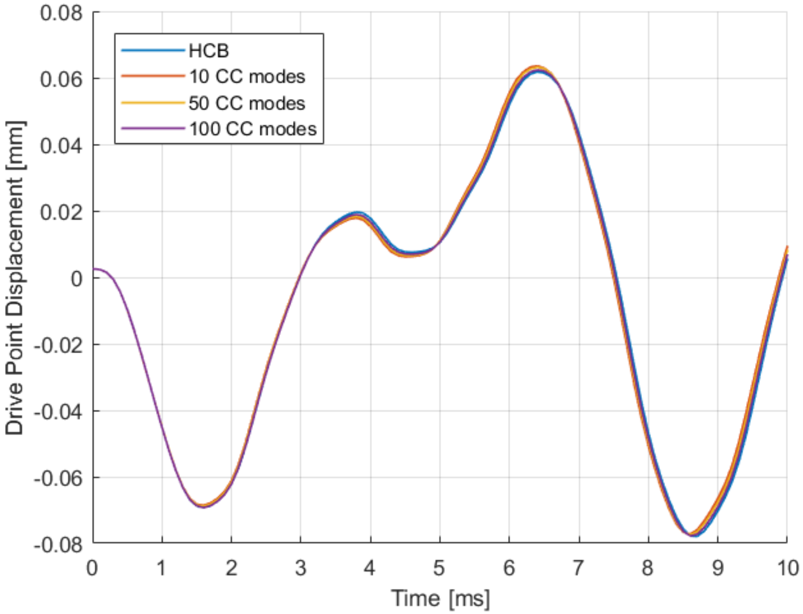


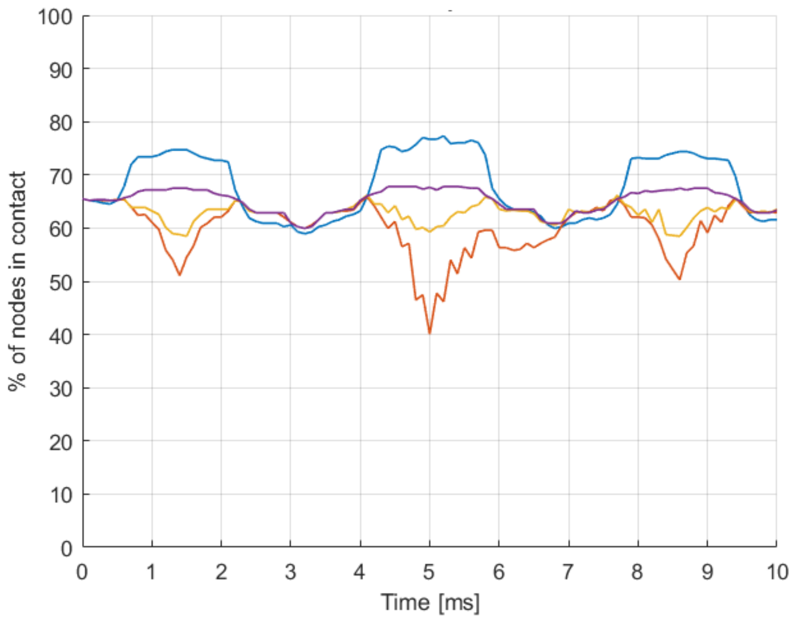


Slowly converges to HCB "truth" solution, but still doesn't allow for loss of contact

Idea: augment with interface RBMs

$$[T_{new}] = [\,[T_{old}]\quad[\Psi_{RBM}]\,]$$





Allows for loss of contact, but still need many CC modes

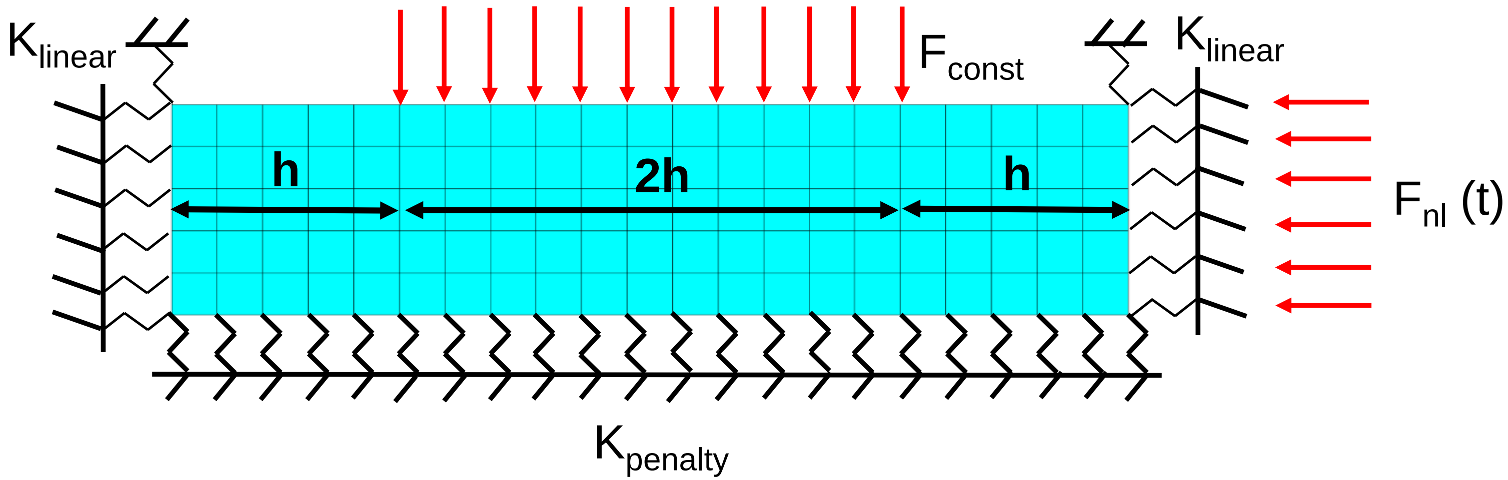
Comparison of Analysis Run Times

Run Time for 10 ms of Simulation Time (Explicit)				
	Hours	Minutes	Seconds	% of HCB Time
HCB	4	20	20	-
SCCe - 10 CC modes	0	38	8	15%
SCCe - 50 CC modes	0	42	8	16%
SCCe - 100 CC modes	1	1	16	24%

Run Time for 10 ms of Simulation Time (Implicit)				
	Hours	Minutes	Seconds	% of HCB Time
HCB	0	4	42	-
SCCe - 10 CC modes	0	6	4	129%
SCCe - 50 CC modes	0	8	23	178%
SCCe - 100 CC modes	0	15	20	326%

Interface reduction here is valuable if you must use explicit methods, but not if implicit methods are available

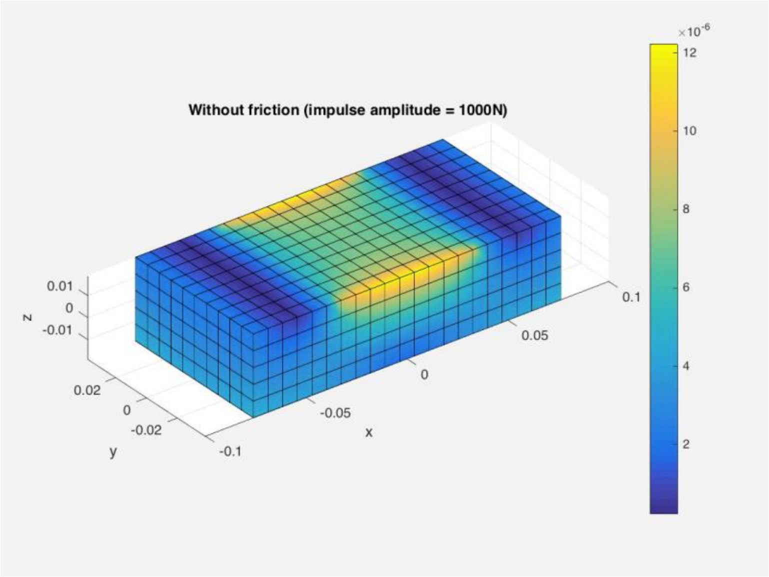
Preliminary results for friction implementation



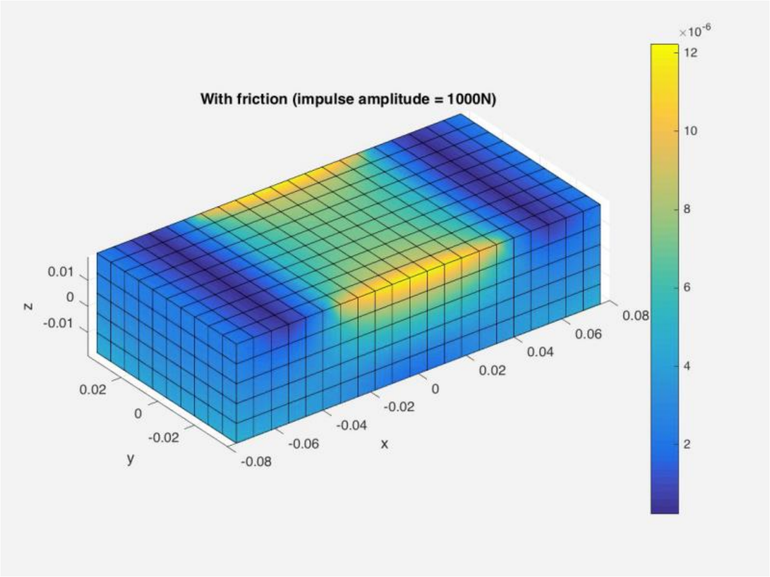
Front view

Friction Model Results

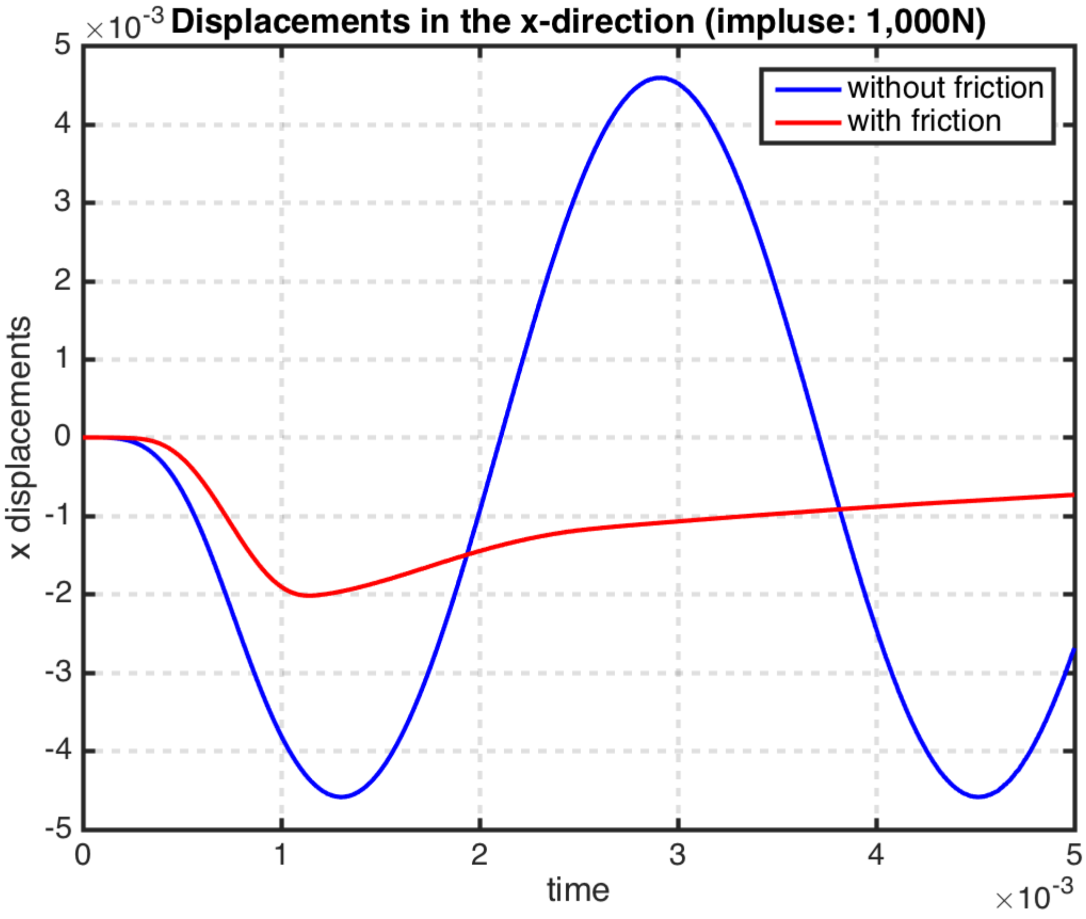
Without friction (impulse amplitude = 1,000N)



With friction (impulse amplitude = 1,000N)

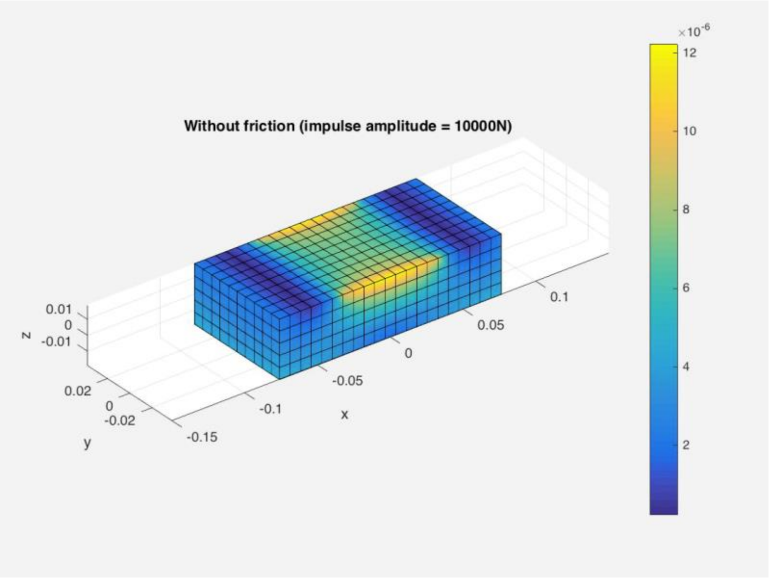


Friction Model Results

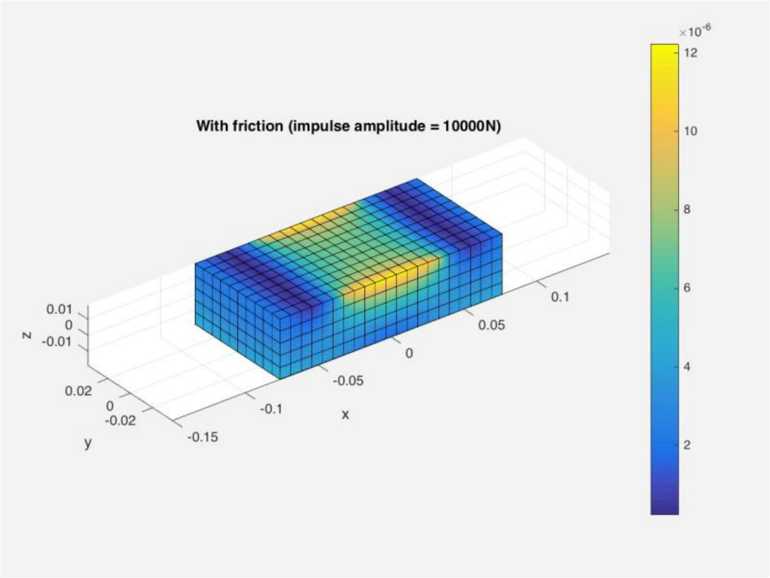


Friction Model Results

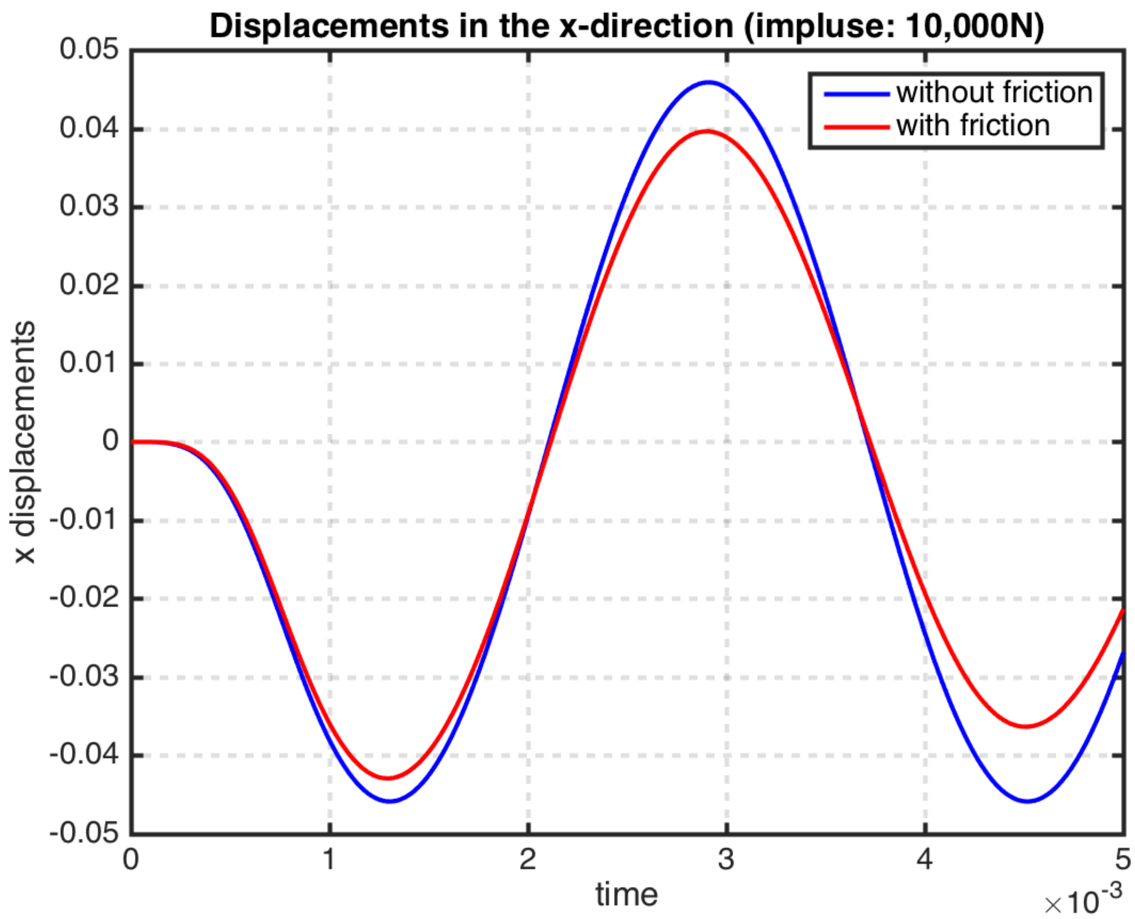
Without friction (impulse amplitude = 10,000N)



With friction (impulse amplitude = 10,000N)



Friction Model Results



Conclusions

- System-level displacement shows agreement between the HCB model and the SCCe model
  - However, SCCe models exhibit difficulties in capturing the contact area
- Interface reduction provides cost savings for explicit time integration scheme (but not implicit)
  - reduces computational cost substantially in dynamic simulations

Next Step

- Incorporate regularized friction elements into the C-Beam model to gain insight into the significance of friction in structural dynamics

Acknowledgments

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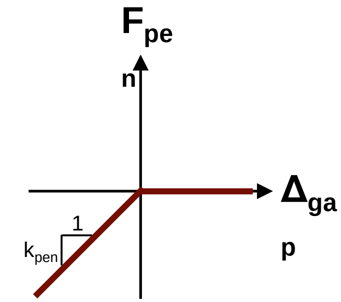
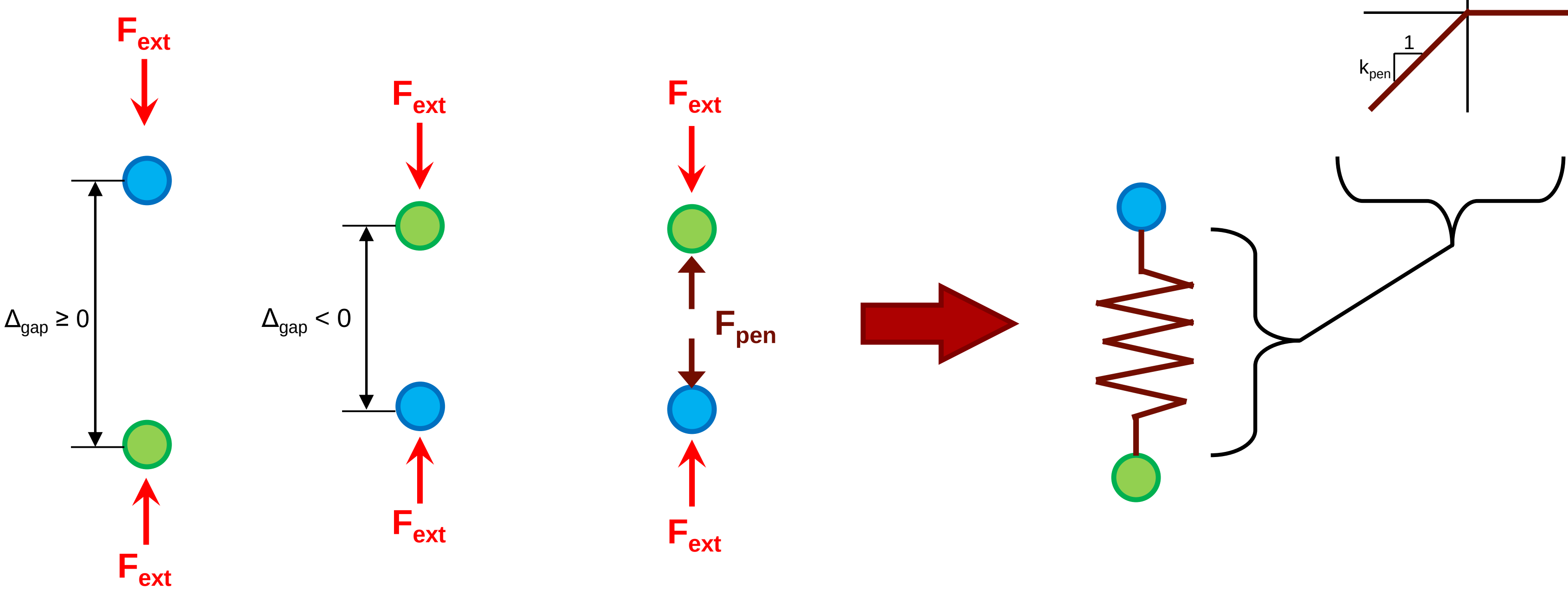
Appendix

Background – Penalty Method





Nodes initially separated

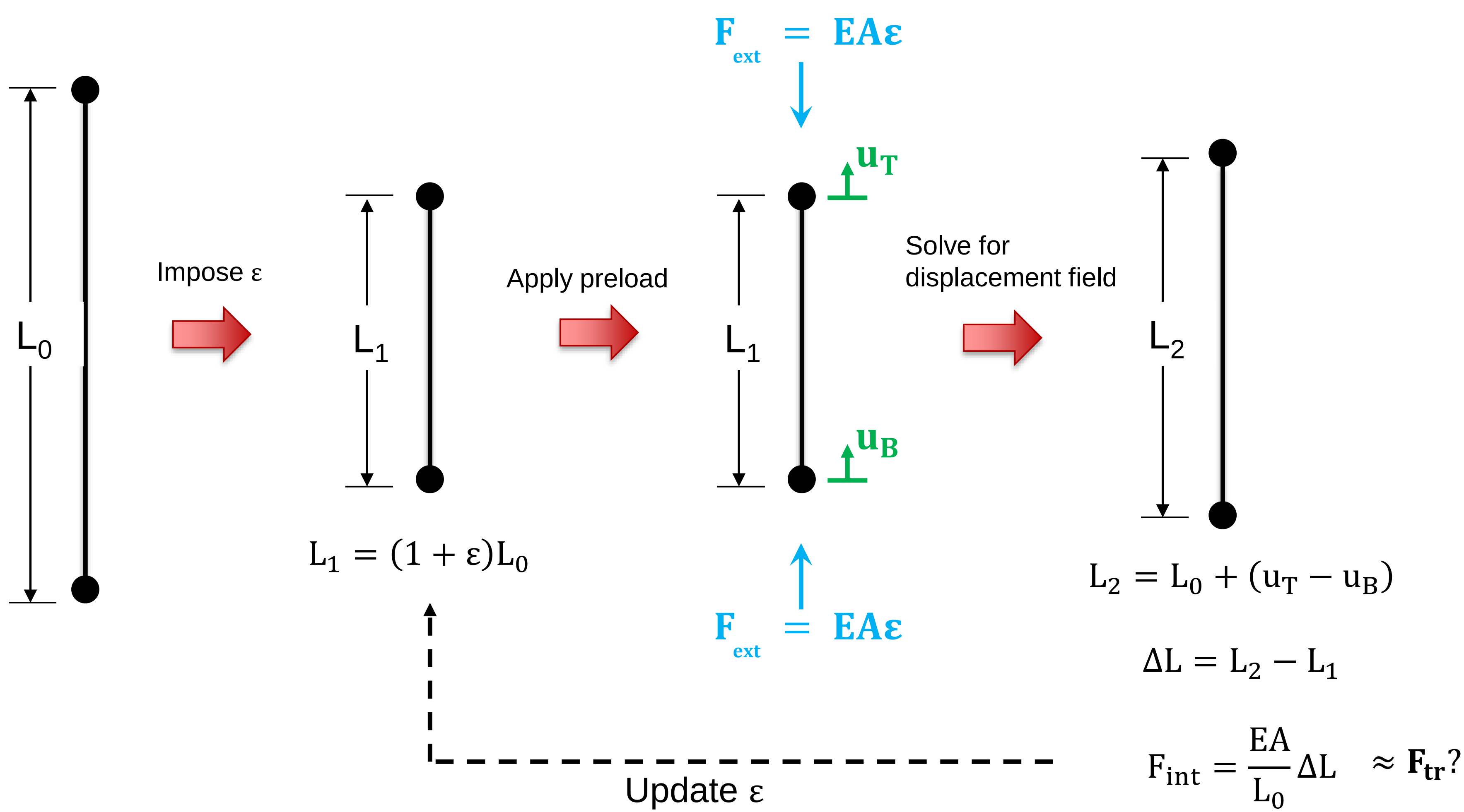
Nodes overlap

Penalty force applied to limit, but not eliminate, overlap

Make penalty force proportional to overlap (constant of proportionality = penalty stiffness)

Violation of the contact condition (nodes do not overlap) is "penalized" by adding energy to the system that is proportional to the non-physical overlap ($E_{pen} = \frac{1}{2} k_{pen} \Delta_{gap}^2$).

What's the correct preload to apply?



What's the correct preload to apply?

- Given bolt torque from experimental group (Project #5), compute transmitted axial force
- Use equation from [1] to do conversion:

$$F_{tr} = \frac{T}{0.159P + 0.578d_2\mu_T + 0.5D_f\mu_H}$$

- $\boldsymbol{F_{tr}}$ = transmitted axial force, $\boldsymbol{T}$ = applied bolt torque, $\boldsymbol{P}$ = bolt pitch, $\boldsymbol{d_2}$ = nominal bolt diameter, $\boldsymbol{D_f}$ = average contact diameter, $\boldsymbol{\mu_T}$ = thread friction coeff., $\boldsymbol{\mu_H}$ = head friction coeff.